Master Internship Proposal for March-September 2026

Toward a unified model of material failure

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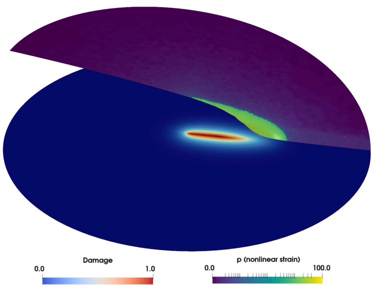
General Context Understanding and predicting material and structure failure is fundamental in many engineering applications and research fields involving solid materials, including nuclear and aerospace industry, biomechanics, or microelectronics. Nonlinear continuum mechanics theories to model structural failure include damage, plasticity, and cohesive crack models. Their goal is to describe the process of accumulation and localization of defects and deformation from their onset to the full structural failure. The key difficulty is that these nonlinear processes couple small material lengthscales (typically micrometres to centimeters) and structural size (from millimeters to kilometers). Moreover, from the mathematical viewpoint, they are associated to the loss of coercitivity of the linearized stiffness operator, and require some form of regularization to avoid spurious localizations and mesh-dependence. In the last 20 years, the variational approach to fracture and the associated phase-field regularized fracture models leaded to significant progress in the theoretical understanding and the numerical simulation of failure phenomena [1]. Phase-field models are based on the variational approximation of brittle fracture models by gradient damage models with an internal length [3-5]. Yet, a satisfactory theory of material and structural failure is still missing. The capability of the available models to correctly predict the crack nucleation and propagation conditions under multiaxial stress states is currently the object of a vivid debate.



Content of the project We have recently begun investigating a novel class of phase-field models to predict the nucleation and propagation of cracks in brittle and ductile materials under complex stress states [6]. Our approach is based on special coupled damage-plasticity models [2]. Preliminary results obtained in the antiplane case suggest that this approach can potentially unify within a single consistent variational theory key concepts developed to predict or prevent material failure: Griffith and cohesive crack models, damage models, plasticity, strength criteria, and limit analysis [6]. The preliminary plan for the internship includes the following steps: (i) theoretical understanding of the properties of the proposed model through one-dimensional analytical solutions, (ii) development of a finite-element numerical implementation of the model using FEniCSx (`https://fenicsproject.org`) for the finite element framework and Mosek (`https://www.mosek.com`) as solver for the plasticity problem, (iii) validation of the model on key test cases. The numerical and theoretical work will include extending the preliminary results obtained for scalar-valued antiplane elasticity to the vectorial case. The internship is intended to be preparatory for a Ph.D. thesis on this subject.

Practical information The internship will take place at the d'Alembert Institute in Jussieu and will be supervised by Corrado Maurini and Marco Moscatelli. The internship may include a research visit period of one to three months at the Department of Mathematics at McMaster University (Canada) . The internship is open to Master 2 students in mechanics, applied mathematics, or computational science. The internship will start in March 2026 and will last for 6 months. Possible funding for the Ph.D. includes scholarships at the Doctoral School of Sorbonne University, research proposals currently under evaluation, and/or co-funding from McMaster University in the framework of a Joint Ph.D.

References

1 Bourdin, B., Francfort, G.A., & Marigo, J.-J. (2008). The Variational Approach to Fracture. *Journal of Elasticity*, 91, 5–148.

2 Alessi, R., Marigo, J.-J., & Vidoli, S. (2014). Gradient damage models coupled with plasticity and nucleation of cohesive cracks. *Archive for Rational Mechanics and Analysis*, 214, 575–615. `https://doi.org/10.1007/s00205-014-0763-8`

3 Tanné, E., Li, T., Bourdin, B., Marigo, J.-J., & Maurini, C. (2018). Crack nucleation in variational phase-field models of brittle fracture. *Journal of the Mechanics and Physics of Solids*, 110, 80–99. `https://doi.org/10.1016/j.jmps.2017.09.006`

4 Zolesi, C., & Maurini, C. (2024). Stability and crack nucleation in variational phase-field models of fracture: Effects of length-scales and stress multi-axiality. *Journal of the Mechanics and Physics of Solids*, 191, 105802. `https://doi.org/10.1016/j.jmps.2024.105802`

5 Vicentini, F., Zolesi, C., Bourdin, B., & Maurini, C. (2025). Variational phase-field modeling of cohesive fracture with flexibly tunable strength surface. *arXiv preprint arXiv:2506.12188*. `https://arxiv.org/abs/2506.12188`

6 Bourdin, B., J.J.Marigo, C.Maurini, C.Zolesi (2025). A variational approach to fracture incorporating volumetric failure. *https://arxiv.org/abs/2506.22558.*